Numerical investigation on radiation characteristics of road traffic noise from semi-underground structure

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ABSTRACT
One of the authors has derived a 2.5-dimensional finite-difference time-domain (FDTD) method by applying Duhamel’s efficient calculation method for a sound field with identical cross-section to 2-dimensional FDTD results. In this paper, as an application of the method, noise radiation characteristics from semi-underground structures were investigated numerically. The validity of the calculation results was confirmed by comparing with 1/20 scale model experimental results. Based on the FDTD calculation results for several cases with typical cross-sectional shapes, an energy-based practical calculation model of road traffic noise in roadside areas of semi-underground structures, which is included in the road traffic noise prediction method “ASJ RTN-Model 2008”, was revised.

1. INTRODUCTION
As a means to mitigate the propagation of road traffic noise, depressed or semi-underground roads are often used in Japan. In such cases, it is difficult to predict the noise propagation from the road structures to the roadside areas, because noise propagation inside the structure is much complicated due to multiple reflections and diffractions. For the aim of prediction of road traffic noise around the road structure, a practical calculation method, in which noise radiation from a semi-underground road is modeled as sound propagation from a hypothetical directional point source, was shown in the ASJ RTN-Model. The calculation method has been developed based on a scale model experiment and the applicable road structures have been limited1,2. On revision of the ASJ RTN-Model, the authors investigated the practical calculation method based on a wave-

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based numerical analysis using the finite-difference time-domain (FDTD) method in order to extend the applicability of the method.

2. NUMERICAL ANALYSIS

A. Semi-Underground Road under Investigation

In this study, a straight semi-underground road with a symmetric cross section was considered as shown in Fig.1. The parameters characterizing this type of road structure are the width of the road, \( R \), width of the mouth, \( W \), height of the underground part of the road structure, \( H \), and the thickness of the overhangs, \( T \). The value of \( H \) was made constant (= 5 m) and sixteen combinations of the other three parameters were chosen, as shown in Table 1. All of the surfaces of the road structure were reflective. As sound sources, two incoherent sources with same sound energy level of 100 dB were assumed to be at center of each driving lane on road surface. Therefore, total sound energy level generated inside the road structure is 103 dB.

![Cross sectional shape of semi-underground road under investigation](image)

**Table 1:** Variation of dimensions of the road structure

<table>
<thead>
<tr>
<th>Case</th>
<th>( R ) (m)</th>
<th>( T ) (m)</th>
<th>( W ) (m)</th>
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</tr>
<tr>
<td>16</td>
<td>30</td>
<td>4</td>
<td>15</td>
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</table>

B. FDTD Analysis

For a 3-D sound field with uniform geometry, a sound pressure response for a point source can be obtained from a 2-D solution using a Fourier-type integration\(^3\). This technique was applied to calculation results obtained from 2-D numerical analysis for cross-section of the semi-underground roads as shown in Fig. 2. A 3-D response at a point \( P \) is calculated from a 2-D solution at a point \( P' \) obtained by the 2-D FDTD analysis. Sound field inside a semi-underground road and scattering field with width of 50 m and height of 25 m were modeled for the analysis as
shown in Fig. 3. Around the scattering field, perfectly matched layers\textsuperscript{4,5} were set in order to realize non-reflecting termination. As FDTD calculation conditions, a discrete spatial grid size and a time step interval were made 0.025 m and 0.01 ms, respectively. In order to examine directivity characteristics of sound radiation from a mouth part of a semi-underground road, 102 receiving points were distributed on a quarter spherical surface of 20 m radius as shown in Fig. 4.

\textbf{C. Analysis of The Calculation Results}

From the calculation results of the 2-D FDTD analysis, 3-D impulse responses at all receiving points were obtained by the 2-D to 3-D transformation\textsuperscript{3}. In the transformation from 2-D to 3-D, the number of FFT points was 524,288 (2\textsuperscript{19}) and consequently the frequency resolution for the integration became 0.19 Hz. Based on the obtained 3-D impulse responses, directivities of sound radiation are examined. In the energy-based practical calculation model proposed in the ASJ RTN-Model, A-weighted sound pressure level is directly calculated. Therefore, in this study, A-weighted single event sound exposure level was calculated from the 3-D impulse response, under the condition where A-weighted sound energy level of a sound source was 100 dB (103 dB by two incoherent sources).

\textbf{D. Comparison with Experimental Result}

The calculated sound radiation characteristics were compared with experimental results obtained with a 1/20 scale model\textsuperscript{2} in order to validate the applicability of the calculation. Figure 5 (a) and 5 (b) show comparison between calculation and experiment for Cases 1 and 3 when a source was located at S\textsubscript{1}, and Fig. 6 (a), 6 (b), show those when two sources were located at S\textsubscript{1} and S\textsubscript{2}. In these figures, single event sound exposure levels on a receiving surface when the sources had a 100 dB sound energy level are shown in a form of net graph. We can see that the radiation directivity varies with the difference of the sectional shape. In all cases, the directivity in x-y plane is sharp in upper and oblique directions, whereas that in the longitudinal section of y-z

\textbf{Figure 2:} Sound field with uniform cross-section in z-direction.

\textbf{Figure 3:} Setting of domain for 2-D FDTD analysis.

\textbf{Figure 4:} Geometrical setting of a semi-underground road, sound sources and receiving points.
plane is rather gentle. Agreement between calculation results and experimental results are fairly good.

3. DETERMINATION OF PARAMETERS IN ASJ RTN-MODEL 2008

Sound radiation characteristics obtained by numerical analysis described above are applied to determination of model parameters in energy-based practical calculation method, “Hypothetical point source method”, included in the ASJ RTN-Model 2008. It should be noted that the expressions for stationary sound source of running road vehicles are described in the ASJ RTN-Model, whereas transient sound source of a unit impulse is dealt with in this study. In this section, the descriptions in the ASJ RTN-Model are translated into those for transient sound source in order to physically correspond with the calculation results performed in this study.

A. Hypothetical point source method specified in the ASJ RTN-Model 2008

Suppose a straight semi-underground road, which has symmetrical cross-section and has equal traffic volumes on both up and down driving lanes, and a receiving point, P, as shown in Fig. 7. When sound sources are positioned at S1 and S2 with total sound energy level of \( L_{A, \text{sh}} \), A-weighted single event sound exposure level at P, \( L_{AE} \), is calculated by following equations as sound propagation from a hypothetical directional sound source, \( S' \), located at a center point of the mouth of the semi-underground road structure.

\[
L_{AE} = L_{A, \text{sh}} + 10 \log \left( a + (1 - a) \cos^{n} \phi \right) - 8 - 20 \log r ,
\]

Figure 5: Comparison between calculation and a scale model experiment for a case where a sound source is positioned at S1.

Figure 6: Comparison between calculation and a scale model experiment for a case where two sound sources are positioned at S1 and S2.
where, $L_{J_{A, su}}$ is apparent sound energy level of a hypothetical sound source of $S'$, $r$ is distance from $S'$ to $P$, and $a$, $n_{\text{max}}$ and $\beta$ are model parameters which represent directivity of sound radiation from the mouth part. $L_{J_{A, su}}$ is calculated as follows.

$$L_{J_{A, su}} = L_{J_{R}} + \Delta L_{\text{dim, su}} + \Delta L_{\text{dir, su}} + \Delta L_{\text{abs, su}},$$

where, $\Delta L_{\text{dim, su}}$, $\Delta L_{\text{dir, su}}$ and $\Delta L_{\text{abs, su}}$ are correction terms regarding the dimensions of the road, directivity of sound radiation and absorbing condition inside the structure, respectively.

B. Sound Energy Level of Hypothetical Sound Source

In the “Hypothetical point source method”, sound energy level of the hypothetical source of $S'$ is assumed to be total sound energy emitting through the mouth part of the semi-underground road. In order to establish the calculation model of sound energy level of $S'$, the total sound energy emitting through the mouth part is discussed from a viewpoint of geometrical acoustics. Now, let us assume following simplifications of sound propagation inside the semi-underground structure and of sound source existing inside the road structure.

**Assumption-1:** Sound sources existing inside the structure (that is, source on each driving lane) are lumped together as an omni-directional point source positioned at a center of the road.

**Assumption-2:** Sound energy arriving to mouth part without reflecting from ceilings and road surface contributes sound radiation to the outside area. That is, only sound reflection paths from sidewalls are considered.

Then, directly arriving sound energy to mouth part, $E_0$, is expressed as follows, when a source with sound energy of $Q$ (sound energy level is made to be $L_{J_{A}}$) exists in a semi-underground structure with width of the road of $R$, height of semi-underground part of $H$ and width of mouth of $W$, as shown in Fig. 8.

$$E_0 = Q \frac{\theta}{\pi} = Q \frac{2}{\pi} \tan^{-1} \frac{W}{2H}. \quad (4)$$

The components of reflected sound can be considered as the sound energy from series of mirror image sources of sidewalls. Figure 9 illustrates contribution from $n$-th order image source. The contribution from the $n$-th order image source, $E_n$, is expressed as,

$$E_n = Q \frac{\theta_n}{\pi}. \quad (5)$$

When $nR >> H$, following relationships hold to a fair approximation.
\[ \frac{W \sin \theta_n}{2} \approx \frac{W}{2} \frac{H}{\sqrt{H^2 + (nR)^2}} = \sqrt{H^2 + (nR)^2} \cdot \frac{\theta_n}{2}, \]  
and using this, sound energy from \( n \)-th mirror image source can be approximated as,

\[ E_n = Q \left( \frac{WH}{\pi R^2 n^2} - \frac{WH^3}{\pi R^4 n^4} \right). \]

Total sound energy radiating from the mouth part is given as a summation of directly arriving energy and contributions from series of mirror images, as follows.

\[ E = E_0 + 2 \sum_{n} E_n = Q \left[ \frac{2}{\pi} \tan^{-1} \frac{W}{2H} + \frac{2WH}{\pi R^2} \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{2WH^3}{\pi R^4} \sum_{n=1}^{\infty} \frac{1}{n^4} \right]. \]

Considering that infinite sums of series which appear in Eq.(8) are \( \sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6 \) and \( \sum_{n=1}^{\infty} \frac{1}{n^4} = \pi^4/90 \),
the total sound energy and its level expression are approximately described by the following equation.

\[ E = Q \left( \frac{2}{\pi} \tan^{-1} \frac{W}{2H} + \frac{\pi WH}{3R^2} - \frac{\pi WH^3}{45R^4} \right). \]

\[ L_{JA,Model} = L_{JA} + 10 \log \left( \frac{2}{\pi} \tan^{-1} \frac{W}{2H} + \frac{\pi WH}{3R^2} - \frac{\pi WH^3}{45R^4} \right). \]
In order to see the validity of this approximation, estimated sound energy calculated by Eq. (10) for all cases was compared with the calculation results obtained by the FDTD method. For calculation of Eq. (10), $L_{JA}$ was made 103 dB. In the numerical analysis based on the FDTD method, single event sound exposure levels at receiving points distributed on hemi-spherical surfaces of 20 m radius were obtained. Therefore, level of total sound energy radiating from the mouth part, $L_{JA,FDTD}$, can be approximated with surface integration of squared value of single event sound exposure. Result of the comparison is shown in Fig. 10. Tendency that the total energy radiating to outside area becomes smaller as the width of the mouth is smaller. Difference between the two values is within 1.3 dB.

C. Parameters Characterizing Directivity

The model parameters of $a$, $n_{max}$ and $\beta$ in Eqs. (1) and (2) are determined by the Least Mean Square method for all cases investigated in this study based on the sound radiation characteristics obtained by the FDTD method. Determined values of the model parameters of $a$, $n_{max}$ and $\beta$ are shown in Fig. 11(a), 11 (b) and 11 (c). In the figures, values of the parameters are arranged as a relationship with open ratio, $W/R$. Regarding the variation of the values of the parameters due to the differences of the dimensions of semi-underground roads, the following tendencies are seen. [1] The values of $a$ are in the range between 0.1 and 0.2. [2] The values of $n_{max}$ become smaller as the open ratio is larger. [3] A rough tendency that the values of $\beta$ become larger as the open ratio is larger and they scatter in the range between 1.6 and 2.7. Based on the result, values of the parameters are assigned as shown in Table. 2 in the ASJ RTN-Model 2008.

![Figure 11: Determined values of the model parameters of $a$, $n_{max}$ and $\beta$.](image)

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<tr>
<th>Case</th>
<th>$a$</th>
<th>$n_{max}$</th>
<th>$\beta$</th>
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<td>1.7</td>
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<tr>
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<tr>
<td>Case 5,6</td>
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<tr>
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<td>1.6</td>
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<td>1.2</td>
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<tr>
<td>Case 16</td>
<td>0.12</td>
<td>1.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
4. INVESTIGATION ON SOUND EXPOSURE LEVEL IN ROADSIDE AREA

In order to see the validity of the energy-based practical calculation model, “Hypothetical point source method”, A-weighted single event sound exposure levels in roadside area were calculated by the method using the values of the parameters shown in Table 2 and the results were compared with the calculation results obtained by the FDTD analysis. Figure 12 shows sound sources and receiving points. Receiving points were distributed on a straight line, which was parallel to the road and was 15 m far from the center of the road. The correspondence between the model and the FDTD analysis is shown in Fig. 13. Averaged difference and standard deviation of the differences were 0.2 dB and 1.5 dB, respectively.

![Figure 12: Sources and receivers for investigation on sound exposure levels in roadside area](image)

![Figure 13: Correspondence between the practical calculation model and the FDTD analysis](image)

4. CONCLUSIONS

Energy-based practical calculation method of road traffic noise in roadside area of a semi-underground road, “Hypothetical point source method” included in the ASJ RTN-Model, was investigated based on the finite-difference time-domain method. On revision of the method, values of parameters characterizing the directional characteristics of the hypothetical point source were determined for several cases and a correction term regarding sound energy of the hypothetical sound source was examined. In this study, semi-underground roads with reflective boundaries were dealt with and noise reduction effects of absorptive treatment were not examined. To extend the applicability of this practical calculation method, it is necessary to make further investigation on the road structure with finite impedance boundary conditions.

REFERENCES